

Sums of three out of four variables equal to a square

Abstract

The below system of equation, has been considered in, volume two of Dickson book (history of theory of numbers). Ajai Choudry in his latest paper (ref # 1), has considered three simultaneous equation with one of them having a maximum degree of three. This paper has considered a system of four equations with a maximum degree of two. The author has provided two methods to arrive at parameterization.

Consider the below system of equation with variables(a,b,c,d):

$$a + b + c = x^2$$

$$a + b + d = y^2$$

$$c + d + a = z^2$$

$$b + c + d = w^2$$

1st method:

we take:

$$a = 3(2n^2 + 4n - m^2 - 2m - 15)$$

$$b = 3(2n^2 + 4n - m^2 + 10m - 15)$$

$$c = 3(2m^2 + 4m - n^2 + 10n - 15)$$

$$d = 3(2m^2 + 4m - n^2 - 2n - 15)$$

Condition is: $(m + n = 4)$

And we get:

$$(x, y, z, w) = [(3n + 3), (3n - 3), (3m - 3), (3m + 3)]$$

where, $(m+n)=4$

Taking $(m,n)=(9,-5)$ we get:

$$(p,q,r,s)=[(-252),(72),(324),(504)]$$

$$\& (x,y,z,w)=(12,18,24,30)$$

2nd method:

We have:

$$a + b + c = x^2$$

$$a + b + d = y^2$$

$$c + d + a = z^2$$

$$b + c + d = w^2$$

we take:

$$a = 3(m^2 - 8m - 6)$$

$$b = 3(m^2 - 2m + 3)$$

$$c = 3(m^2 + 4m + 6)$$

$$d = 3(m^2 + 10m + 3)$$

& we get:

$$(x, y, z, w) = [(3m - 3), (3m), (3m + 3), (3m + 6)]$$

For $m=13$ we get,

$$(a,b,c,d)=[177,438,681,906]$$

$$(x,y,z,w)=(36,39,42,45)$$

Conclusion:

There is a system of six equations in four variables which can be considered by viewers for parametrization & is shown below:

$[(ab + 1), (ac + 1), (bc + 1), (ad + 1), (bd + 1), (cd + 1)]$ are all to be made squares & represented as, (p, q, r, s, t, u)

The numerical solution to the above is:

$$(a, b, c, d) = (1, 3, 8, 120) \text{ \& } (p, q, r, s, t, u) = (2, 3, 5, 11, 19, 31)$$

References

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Three integers whose sum, product and the sum of the products of the integers, taken two at a time, are perfect squares.

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4) Published paper, Oliver Couto,

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5) Ajai Choudhry, Number theory web site,

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